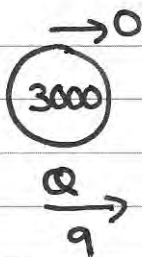
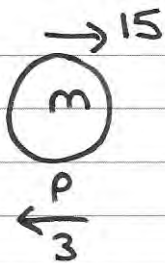


11 JAN 12

1. A railway truck  $P$ , of mass  $m$  kg, is moving along a straight horizontal track at  $15 \text{ m s}^{-1}$ . Truck  $P$  collides with a truck  $Q$  of mass  $3000$  kg, which is at rest on the track. Immediately after the collision the speed of  $P$  is  $3 \text{ m s}^{-1}$  and the speed of  $Q$  is  $9 \text{ m s}^{-1}$ . The direction of motion of  $P$  is reversed by the collision.

Modelling the trucks as particles, find

- (a) the magnitude of the impulse exerted by  $P$  on  $Q$ , (2)
- (b) the value of  $m$ . (3)



$$\text{Mom } Q \text{ before} = 0$$

$$\text{Mom } Q \text{ after} = 27000$$

$$\Rightarrow \text{Impulse} = \underline{27000 \text{ Ns}}$$

b) Conservation of momentum  $\Rightarrow$

$$15m = -3m + 27000 \Rightarrow 18m = 27000$$

$$\Rightarrow m = \underline{1500 \text{ kg}}$$



2. A car of mass 1000 kg is towing a caravan of mass 750 kg along a road. The caravan is connected to the car by a tow-bar which is parallel to the direction of motion of the car and the caravan. The tow-bar is modelled as a light rod. The engine of the car provides a constant driving force of 3200 N. The resistances to the motion of the car and the caravan are modelled as constant forces of magnitude 800 newtons and  $R$  newtons respectively.

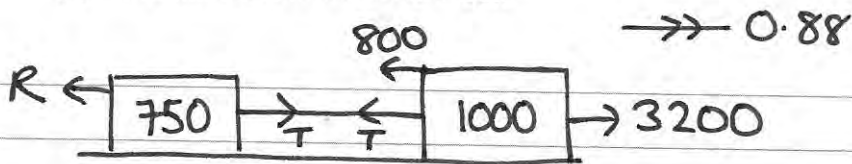
Given that the acceleration of the car and the caravan is  $0.88 \text{ ms}^{-2}$ ,

- (a) show that  $R = 860$ ,

(3)

- (b) find the tension in the tow-bar.

(3)



a) whole system  $\vec{R}F = ma$

$$3200 + T - T - 800 - R = (750 + 1000) \times 0.88$$

$$\Rightarrow 2400 - R = 1540 \Rightarrow R = 2400 - 1540 = \underline{860 \text{ N}}$$

b) Caravan  $\vec{R}F = ma$

$$T - 860 = 750 \times 0.88 \Rightarrow T = \underline{1520 \text{ N}}$$

3. Three forces  $F_1$ ,  $F_2$  and  $F_3$  acting on a particle  $P$  are given by

$$F_1 = (7i - 9j) \text{ N}$$

$$F_2 = (5i + 6j) \text{ N}$$

$$F_3 = (pi + qj) \text{ N}$$

where  $p$  and  $q$  are constants.

Given that  $P$  is in equilibrium,

(a) find the value of  $p$  and the value of  $q$ .

(3)

The force  $F_3$  is now removed. The resultant of  $F_1$  and  $F_2$  is  $R$ . Find

(b) the magnitude of  $R$ ,

(2)

(c) the angle, to the nearest degree, that the direction of  $R$  makes with  $j$ .

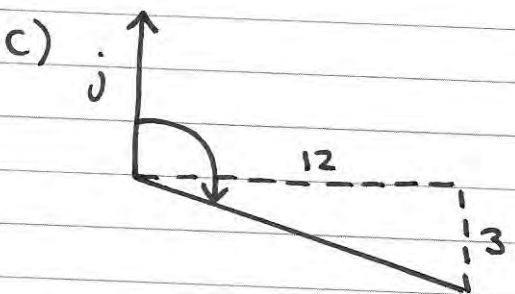
(3)

$$a) F_1 + F_2 + F_3 = 0 \Rightarrow \begin{pmatrix} 7 \\ -9 \end{pmatrix} + \begin{pmatrix} 5 \\ 6 \end{pmatrix} + \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow p = -12, q = 3$$

$$b) R_f = \begin{pmatrix} 7 \\ -9 \end{pmatrix} + \begin{pmatrix} 5 \\ 6 \end{pmatrix} = \begin{pmatrix} 12 \\ -3 \end{pmatrix} \quad |R| = \sqrt{12^2 + 3^2}$$

$$|R| = 12.4 \text{ N (3sf)}$$



$$\text{angle} = 90 + \tan^{-1}\left(\frac{3}{12}\right)$$

$$= \underline{104^\circ} \text{ (n.d.)}$$

4.

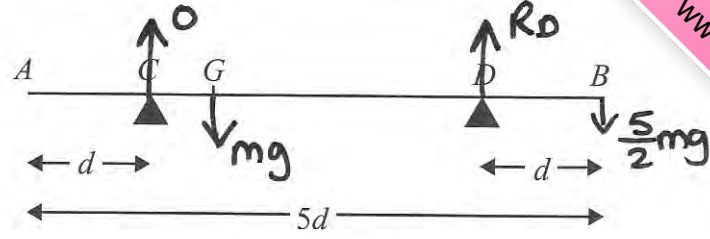


Figure 1

A non-uniform rod  $AB$ , of mass  $m$  and length  $5d$ , rests horizontally in equilibrium on two supports at  $C$  and  $D$ , where  $AC = DB = d$ , as shown in Figure 1. The centre of mass of the rod is at the point  $G$ . A particle of mass  $\frac{5}{2}m$  is placed on the rod at  $B$  and the rod is on the point of tipping about  $D$ .

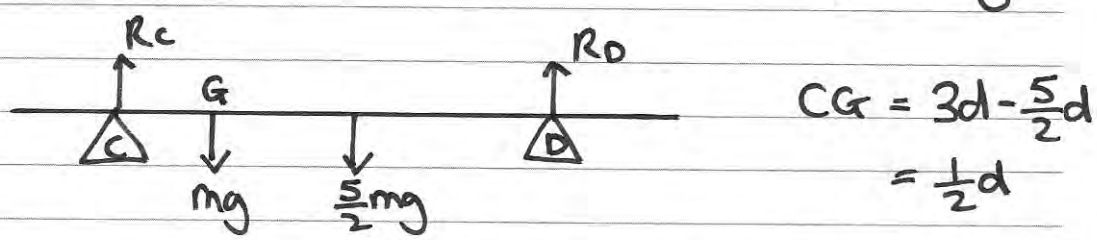
(a) Show that  $GD = \frac{5}{2}d$ . ? (4)

The particle is moved from  $B$  to the mid-point of the rod and the rod remains in equilibrium.

(b) Find the magnitude of the normal reaction between the support at  $D$  and the rod. (5) ?

a)  $\curvearrowright \frac{5}{2}mg \times d = mg \times GD \Rightarrow GD = \frac{5}{2}d$

b)  $R_C \uparrow = 0 \Rightarrow R_D = mg + \frac{5}{2}mg = \frac{7}{2}mg$  read the question ;



$\curvearrowright mg \times \frac{1}{2}d + \frac{5}{2}mg \times \frac{3}{2}d = R_D \times 3d$

$\frac{1}{2}mgd + \frac{15}{4}mgd = R_D \times 3d$

$\frac{17}{4}mgd = R_D \times 3d \Rightarrow R_D = \frac{17}{12}mg$

5. A stone is projected vertically upwards from a point  $A$  with speed  $u$  m/s. The stone moves freely under gravity until it returns to  $A$ . The time between the stone is projected and the instant that it returns to  $A$  is  $3\frac{4}{7}$  seconds.

Modelling the stone as a particle,

- (a) show that  $u = 17\frac{1}{2}$ , (3)
- (b) find the greatest height above  $A$  reached by the stone, (2)
- (c) find the length of time for which the stone is at least  $6\frac{3}{5}$  m above  $A$ . (6)

a)  $s = 0$   $s = ut + \frac{1}{2}at^2$   
 $u$   
 $v$   $0 = 3\frac{4}{7}u - 4.9(3\frac{4}{7})^2$   
 $a = -9.8$   $3\frac{4}{7}u = 4.9(3\frac{4}{7})^2$   
 $t = 3\frac{4}{7}$   $u = 4.9 \times 3\frac{4}{7} = 17.5 \#$

b)  $s$   $v^2 = u^2 + 2as$   
 $u = 17.5$   $0 = 17.5^2 - 19.6s$   
 $v = 0$   $\Rightarrow s = 15.625 \approx 15.6 \text{ m (3sf)}$   
 $a = -9.8$   
 $t$

c)  $s = 6.6$   $s = ut + \frac{1}{2}at^2$   
 $u = 17.5$   $6.6 = 17.5t - 4.9t^2$   
 $v$   $4.9t^2 - 17.5t + 6.6 = 0$   
 $a = -9.8$   $49t^2 - 175t + 66 = 0$   
 $t$   $(7t-3)(7t-22) = 0$   
 $t_1 = \frac{3}{7}$   $t_2 = \frac{22}{7}$

time above =  $\frac{19}{7}$  sec

6. A car moves along a straight horizontal road from a point  $A$  to a point  $B$ . The car accelerates from rest at  $A$  to a speed of  $15 \text{ m s}^{-1}$  at a constant rate. The time for which the car accelerates is  $\frac{1}{3}T$  seconds. The car maintains  $15 \text{ m s}^{-1}$  for  $T$  seconds. The car then decelerates at a constant rate of  $2.5 \text{ m s}^{-2}$  stopping at  $B$ .

(a) Find the time for which the car decelerates.

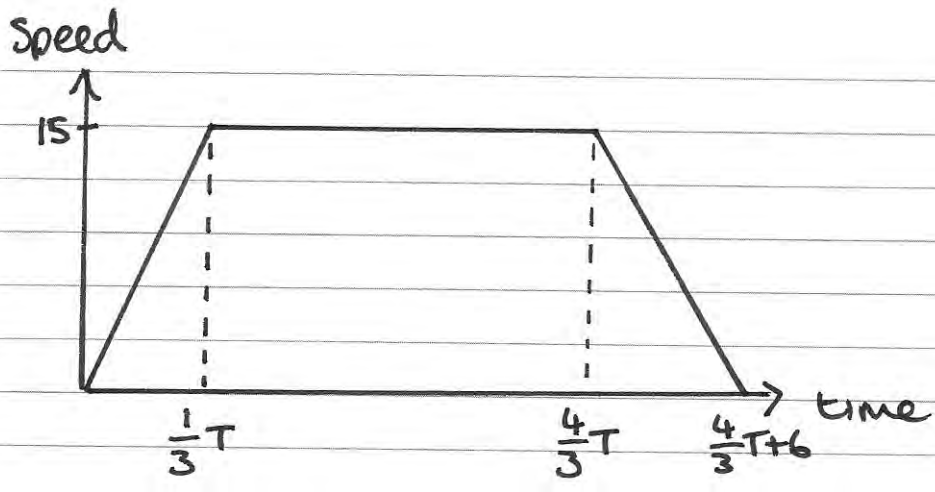
$$\frac{15}{2.5} = 6 \text{ sec} \quad (2)$$

(b) Sketch a speed-time graph for the motion of the car. (2)

(c) Find the value of  $T$ . (4)

(d) Find the value of  $a$ . (2)

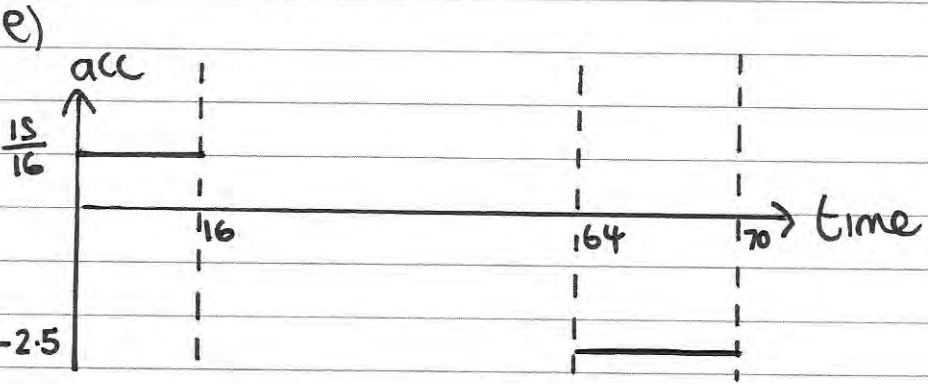
(e) Sketch an acceleration-time graph for the motion of the car. (3)



$$c) \left(\frac{1}{6}T \times 15\right) + (T \times 15) + (3 \times 15) = 885$$

$$17\frac{1}{2}T = 840 \Rightarrow T = \underline{48 \text{ sec}}$$

$$d) a = \frac{15}{16} \text{ m s}^{-2}$$



7. [In this question, the unit vectors  $\mathbf{i}$  and  $\mathbf{j}$  are due east and due north respectively, relative to a fixed origin  $O$ .]

A boat  $P$  is moving with constant velocity  $(-4\mathbf{i} + 8\mathbf{j})$  km h<sup>-1</sup>.

- (a) Calculate the speed of  $P$ .

$$\sqrt{4^2 + 8^2} = \underline{8.94 \text{ km/h (3sf)}} \quad (2)$$

When  $t = 0$ , the boat  $P$  has position vector  $(2\mathbf{i} - 8\mathbf{j})$  km. At time  $t$  hours, the position vector of  $P$  is  $\mathbf{p}$  km.

- (b) Write down  $\mathbf{p}$  in terms of  $t$ .
- $$\mathbf{p} = \begin{pmatrix} 2 \\ -8 \end{pmatrix} + t \begin{pmatrix} -4 \\ 8 \end{pmatrix} = \begin{pmatrix} 2 - 4t \\ 8t - 8 \end{pmatrix} \quad (1)$$

A second boat  $Q$  is also moving with constant velocity. At time  $t$  hours, the position vector of  $Q$  is  $\mathbf{q}$  km, where

$$\mathbf{q} = 18\mathbf{i} + 12\mathbf{j} - t(6\mathbf{i} + 8\mathbf{j}) = \begin{pmatrix} 18 - 6t \\ 12 - 8t \end{pmatrix}$$

Find

- (c) the value of  $t$  when  $P$  is due west of  $Q$ , (3)
- (d) the distance between  $P$  and  $Q$  when  $P$  is due west of  $Q$ . (3)

c) due west  $\Rightarrow$  j component equal

$$8t - 8 = 12 - 8t \Rightarrow 16t = 20 \quad t = \frac{5}{4}$$

d) when  $t = \frac{5}{4}$  i component of  $\mathbf{p} = 2 - 4\left(\frac{5}{4}\right) = -3$

j component of  $\mathbf{q} = 12 - 8\left(\frac{5}{4}\right) = 10 - 5 = 5$

$\therefore$  distance between  $P$  and  $Q = \underline{13.5 \text{ km}}$

8.

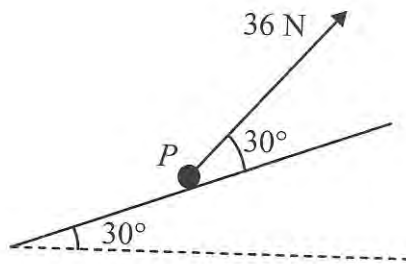


Figure 2

A particle  $P$  of mass  $4\text{ kg}$  is moving up a fixed rough plane at a constant speed of  $16\text{ m s}^{-1}$  under the action of a force of magnitude  $36\text{ N}$ . The plane is inclined at  $30^\circ$  to the horizontal. The force acts in the vertical plane containing the line of greatest slope of the plane through  $P$ , and acts at  $30^\circ$  to the inclined plane, as shown in Figure 2. The coefficient of friction between  $P$  and the plane is  $\mu$ . Find

(a) the magnitude of the normal reaction between  $P$  and the plane,

(4)

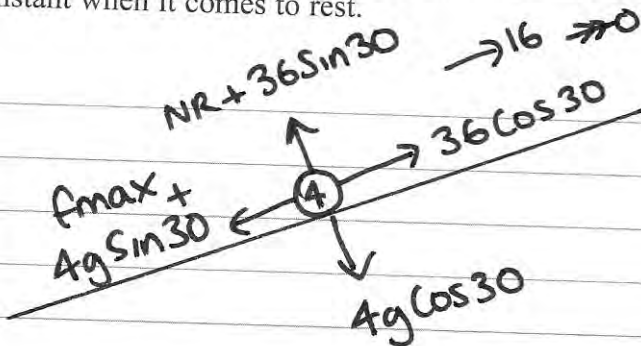
(b) the value of  $\mu$ .

(5)

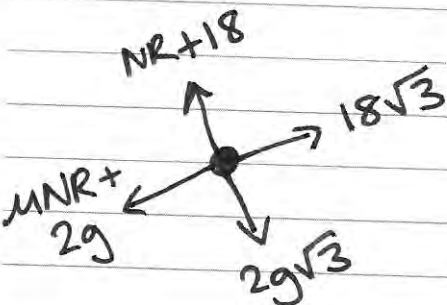
The force of magnitude  $36\text{ N}$  is removed.

(c) Find the distance that  $P$  travels between the instant when the force is removed and the instant when it comes to rest.

(5)



Constant speed  
 $\Rightarrow \text{acc} = 0$   
 $\Rightarrow \text{equilibrium}$



$$R \uparrow = 0 \Rightarrow NR = -18 + 2\sqrt{3}g$$

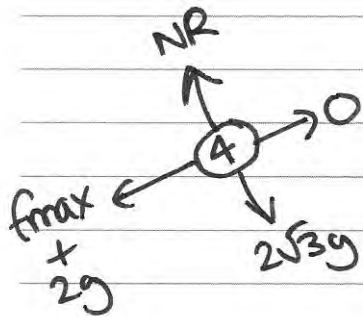
$$= 15.9\text{ N (3sf)}$$

$$R \nearrow = 0 \Rightarrow \mu NR = 18\sqrt{3} - 2g$$

$$\therefore \mu = \frac{18\sqrt{3} - 2g}{-18 + 2\sqrt{3}g} \approx 0.726$$

$$(3\text{sf})$$





$$NR = 2\sqrt{3}g$$

$$f_{max} = \mu \times 2\sqrt{3}g = 24$$

$$Rf = ma$$

$$-24.6432... - 2g = 4a$$

$$\Rightarrow a = -11.0608...$$

$S$

$$u = 16$$

$$v = 0$$

$$a = -11.0608...$$

$t$

$$v^2 = u^2 + 2as$$

$$0 = 256 - 22.1216... S$$

$$\therefore S \approx \underline{11.6 \text{ m}} \quad (3sf)$$